# Geometrical Model for Point-to-Point Multi-Polarized Massive MIMO Systems

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***Abstract*—In massive MIMO systems, a large number of antennas are difficult to be placed in a limited space, and the antenna space limitation causes a high spatial correlation between the antennas, furthering causes systems performance degradation. In this paper, we implement the multi-polarized antennas in point-to-point massive MIMO systems to reduce the correlation between antennas to enhance the systems performance and realize the space efficiency. Also we establish a 3-D geometrical channel model for the proposed point-to-point multi-polarized massive MIMO systems. The channel is modeled as a Ricean fading channel and the average correlation for the whole systems is defined to indicate the correlation degree of the systems. We compare the performance of multi-polarized massive MIMO systems with uni-polarized massive MIMO systems in different communication scenarios. The achieved results demonstrate that the multi-polarized massive MIMO systems have better performance compared to the uni-polarized massive MIMO systems in many situations.**

***Index Terms*—Massive MIMO; spatial correlation; channel model.**

## I. Introduction

Massive multiple-input multiple-output (MIMO) systems have been regarded as a candidate technology for the 5th generation (5G) cellular networks because of their advantages in terms of high data rate, enhanced reliability and high energy efficiency [1] [2]. Unlike conventional MIMO systems with small and compact antenna arrays, massive MIMO systems have a base station (BS) equipped with a large number of antennas (tens or hundreds of antennas). A large number of antennas are difficult to be placed in a limited space. If they can be deployed, the high spatial correlation between the antennas will cause system performance degradation because of the space and size restrictions [3]. In the traditional MIMO systems, the multi-polarized antennas have been implemented to reduce the correlation between antennas to improve the systems performance and realize the space efficiency [4].

Most of existing works [5] [6] [7] on developing massive MIMO systems are based on the assumption that the channels are independent and uncorrelated Rayleigh fading channels. In reality, this assumption is very difficult to realize when the number of multiple antennas is very large [8]. Few works [9] [10] care about the correlation between antennas in the massive MIMO systems which actually is important to the systems performance. Therefore, in this paper, based on our previous work [11], we first implement the multi-polarized antennas in point-to-point massive MIMO systems to reduce the correlation between antennas to enhance the systems performance and realize the space efficiency. Then we establish a 3-D geometrical channel model for the point-to-point multi-polarized massive MIMO systems and focus on the systems performances, which seem not to be done yet. The channel is modeled as a Ricean fading channel including a fixed (Line of Sight, LoS) part and a scattering (Non-Line of Sight, NLoS) part. Both the azimuth angles of arrival (AAoAs) and elevation angles of arrival (EAoAs) have been taken into account. The proposed model has parameters of communication environment and antennas such as environment scattering status, XPD, antenna spacing and can be adjusted according to the communication environment compared to the uncorrelated Rayleigh fading channels and the traditional Rician MIMO channels [12]. We compare the performance of multi-polarized massive MIMO systems with uni-polarized massive MIMO systems in different scenarios. It is found that the multi-polarized massive MIMO systems have better performance compared to the uni-polarized massive MIMO systems in many situations.

## II. 3-D Channel Modeling of Multi-Polarized Massive MIMO Systems

### A. Correlation Analysis of Multi-Polarized Massive MIMO Systems

The polarization orientation of electromagnetic waves will change after passing through the wireless channels because of the reflections, diffractions and scatterings of waves in real wireless communication channels. This phenomenon is called channel depolarization and a commonly used method for describing channel depolarization is to define the cross-polarization discrimination (XPD) [13]

$$XPD = \frac{\mathrm{E}\{|h_{VV}|^2\}}{\mathrm{E}\{|h_{HV}|^2\}} = \frac{\mathrm{E}\{|h_{HH}|^2\}}{\mathrm{E}\{|h_{VH}|^2\}} = \frac{1-a}{a}, a = \frac{1}{XPD+1} \tag{1}$$

where $h_{XY}(X, Y \in V, H)$ is the $XY$ channel, and E{} represents the expectation operator. $a(0 < a \leq 1)$ is defined for the convenience of modeling and computing. $a$ is directly related to the $XPD$ and corresponds to the part of the power that is coupled from $X$ polarization to $Y$ polarization [14].

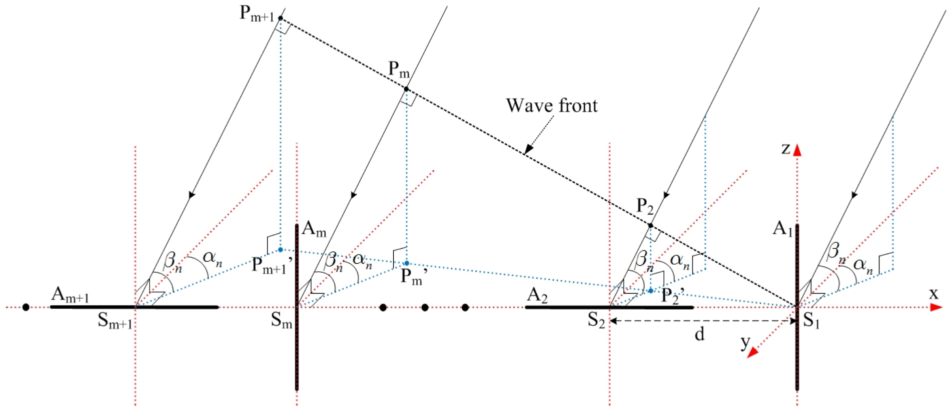



Fig. 1. A 3-D point-to-point multi-polarized massive MIMO system transmission scenario at one link terminal.

A 3-D point-to-point multi-polarized massive MIMO system transmission scenario at one link terminal is shown in Fig. 1. Usually, we regard the ground as the reference, which means the antennas perpendicular to the ground are vertically polarized antennas, and the antennas parallel to the ground are horizontally polarized antennas [14]. Moreover, an antenna is designed to receive a signal with a certain polarization, and it is completely isolated to the cross-polarization component (i.e., the vertically polarized antennas have zero gain to the horizontally polarized direction signal and vice versa) [14]. In Fig. 1, the $x$-$S_1$-$y$ plane is the horizontal plane. Hence, antennas $A_1$ and $A_m$ are vertically polarized antennas, while antennas $A_2$ and $A_{m+1}$ are horizontally polarized antennas, and we make vertically polarized antennas and horizontally polarized antennas be arranged alternately at two link terminals in our proposed point-to-point multi-polarized massive MIMO system. The antennas are in the far-field of the signals, therefore, the wave fronts of signals at antennas are plane wave. The multipath signals come from arbitrary direction with AAoAs $\alpha_n (n \in \{1, 2...N\})$ and EAoAs $\beta_n (n \in \{1, 2...N\})$ for all the antennas, and every antenna has the same AAoAs and EAoAs distributions. The adjacent antenna spacing is $d$. For antenna $A_2$, $S_2P_2^{'}$ is the projection of $S_2P_2$ on the horizontal plane. $\angle P_2S_2P_2^{'}$ is $\beta_n$, and $\angle P_2^{'}S_2S_1$ equals to $(\pi/2 - \alpha_n)$. Therefore, the angle $\angle P_2S_2S_1$ is given by [11]

$$\begin{aligned} cos\angle P_2S_2S_1 &= cos\angle P_2^{'}S_2S_1 cos\angle P_2S_2P_2^{'} \\ &= cos(\pi/2 - \alpha_n)cos(\beta_n). \end{aligned} \tag{2}$$

The transmission distance difference between antennas $A_1$ and $A_2$ is $P_2S_2$, and the corresponding time delay difference is $P_2S_2$/c=d$cos(\pi/2-\alpha_n)cos(\beta_n)$/c. Regarding antenna $A_1$ as a reference antenna, the channel impulse responses of antennas $A_1$ and $A_2$ can be expressed as

$$h_1 = \sum_{n=1}^{N} \sqrt{P_{V,n}(1-a) + P_{H,n}a} e^{j\phi_n} \tag{3}$$

$$\begin{aligned} h_2 = &\sum_{n=1}^{N} \sqrt{P_{H,n}(1-a) + P_{V,n}a} \\ &\times e^{j(\phi_n + 2\pi dcos(\frac{\pi}{2}-\alpha_n)cos(\beta_n)/\lambda)} \end{aligned} \tag{4}$$

where $P_{X,n}$ is the $n$th path $X$ polarized power and $\phi_n$ is the $n$th path phase. $P_{X,n}(1-a)$ is the $n$th path power maintain in the co-polarization and $P_{X,n}a$ is the $n$th path power leakage to the cross-polarization at the antennas [15]. $\lambda$ is the carrier wavelength. In massive MIMO systems, all the antennas are uniformly-spaced [16]. Therefore, for antenna $A_m$, the channel impulse response is

$$\begin{aligned} h_m = &\sum_{n=1}^{N} \sqrt{P_{V,n}(1-a) + P_{H,n}a} \\ &\times e^{j(\phi_n + 2\pi d(m-1)cos(\frac{\pi}{2}-\alpha_n)cos(\beta_n)/\lambda)}. \end{aligned} \tag{5}$$

Then the spatial correlation function between antennas $A_1$ and $A_2$ is defined as $\rho_{1,2}$

$$\begin{aligned} \rho_{1,2} &= E\{h_1 h_2^*\} \\ &= E\{\sum_{n=1}^{N} \sqrt{[P_{V,n}(1-a) + P_{H,n}a][P_{H,n}(1-a) + P_{V,n}a]} \\ &\times e^{-j2\pi dcos(\frac{\pi}{2}-\alpha_n)cos(\beta_n)/\lambda}\}. \end{aligned} \tag{6}$$

As $N \to \infty$, discrete AAoAs $\alpha_n$ and discrete EAoAs $\beta_n$ can be replaced with continuous random variables $\alpha$ and $\beta$ having a joint probability density function (pdf) $p(\alpha, \beta)$ [17]. We assume that AAoAs and EAoAs are independent of each other, then we have $p(\alpha, \beta) = p(\alpha)p(\beta)$. Also, the discrete power $P_{X,n}$ can be also replaced with power variable $P_X$. Then the $\rho_{1,2}$ can be written as

$$\begin{aligned} \rho_{1,2} = &\sqrt{[P_V(1-a) + P_H a][P_H(1-a) + P_V a]} \\ &\times \int\int e^{-j2\pi dcos(\frac{\pi}{2}-\alpha)cos(\beta)/\lambda} p(\alpha)p(\beta)d\beta d\alpha. \end{aligned} \tag{7}$$

There are many different distributions to characterize the AAoAs distribution, such as Uniform, Gaussian, Laplacian. Here we use the uniform distribution with certain angle spread

(AS) to characterize the AAoAs distribution especially in the future 5G small cell communication scenarios, which is defined as

$$p(\alpha) = \frac{1}{2\Delta\alpha}, -\Delta\alpha + \alpha_0 \le \alpha \le \Delta\alpha + \alpha_0 \tag{8}$$

where $\alpha_0$ is the mean AAoAs, and $\Delta\alpha$ is the AS. For the EAoAs distribution, we use the cosine pdf [17]

$$p(\beta) = \frac{\pi}{4\beta_{max}} \cos(\frac{\pi}{2}\frac{\beta}{\beta_{max}}), |\beta| \le \beta_{max} \le \frac{\pi}{2} \tag{9}$$

where $\beta_{max}$ is the maximum EAoA. Then we can derive the spatial correlation between any two antennas.

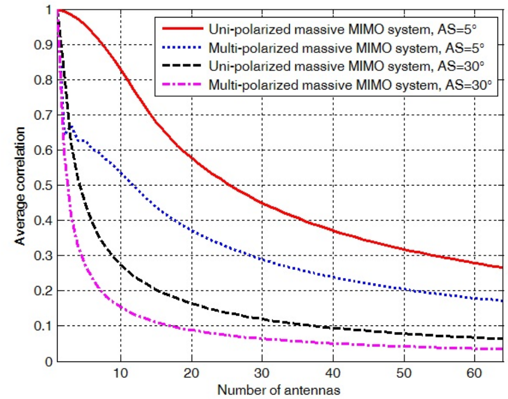
Since the massive MIMO systems have tens or hundreds of antennas, we define the average correlation for the whole systems to indicate the correlation degree of the systems. First we define the average correlation for a certain single antenna. We assume that the whole massive MIMO system has $M$ antennas at one link terminal, and for antenna $A_m$, the average correlation $\bar{\rho}_m$ is defined as

$$\begin{aligned}\bar{\rho}_m &= \frac{\rho_{m,1} + \rho_{m,2} + ... + \rho_{m,m} + ... + \rho_{m,M}}{M} \\ &= \frac{\sum_{i=1}^{M} \rho_{m,i}}{M}.\end{aligned} \tag{10}$$

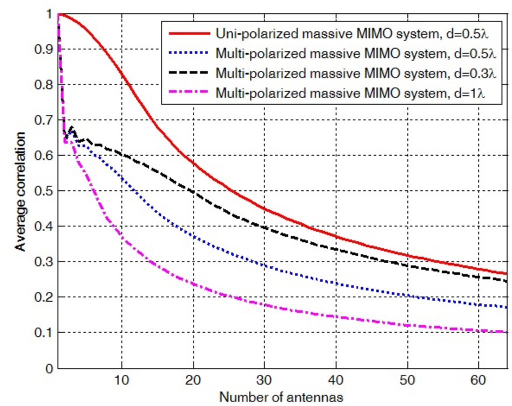
Then the average correlation of the whole systems is actually the mean value of all the average correlations of single antenna

$$\begin{aligned}\bar{\rho} = &[(\rho_{1,1} + \rho_{1,2} + ... + \rho_{1,M}) + (\rho_{2,1} + \rho_{2,2} + ... + \rho_{2,M}) \\ &... + (\rho_{M,1} + \rho_{M,2} + ... + \rho_{M,M})]/M^2 \\ &= \frac{\sum_{i=1}^{M} \bar{\rho}_i}{M}.\end{aligned} \tag{11}$$

Fig. 2 (a) shows that the average correlation of both uni-polarized and multi-polarized massive MIMO systems vary as the number of antennas increases with different AS. The antenna spacing is set to be $\lambda/2$, $\beta_{max}$ is equal to 30° and XPD is equal to 10 dB. Both the mean AAoAs and mean EAoAs are set to be 0° and the power is normalized. From Fig. 2(a) we can see that the average correlation decreases as the number of antennas increases. Larger AS (rich scattering) results in a lower average correlation. The multi-polarized massive MIMO systems have better performance compared to the uni-polarized massive MIMO systems especially in a poor scattering status (small AS). In addition to the scattering environment, the average correlation is also sensitive to the antenna spacing. Fig. 2 (b) draws the average correlation varying with different antenna spacing, and the AS is set to be 5°. From which we can see that the larger antenna spacing results in a lower average correlation. Also the multi-polarized massive MIMO systems can obtain a very low average correlation compared with the uni-polarized massive MIMO systems even with smaller antenna spacing. Therefore, using multi-polarized antennas in massive MIMO systems can help to reduce the correlation between antennas and can reduce the demand for large antenna spacing to realize the space




(a)




(b)

Fig. 2. (a) Average correlation versus number of antennas with different AS (b) Average correlation versus number of antennas with different antenna spacing.

efficiency. Fig. 3 shows that the average correlation of multi-polarized massive MIMO systems is also sensitive to the XPD. A higher XPD can result in a lower average correlation. This is because as the XPD increases, more power will maintain in the co-polarization, which reduces the coupling effect between different polarized antennas.

### *B. Channel Capacity Analysis of Multi-Polarized Massive MIMO Systems*

We consider a point-to-point massive MIMO system with a transmitter equipped with $M$ antennas and a receiver with $K$ antennas. Usually, wireless channel can be modeled as a Ricean fading channel, which means the channel matrix is composed of a fixed (LoS) part and a scattering (NLoS) part according to [4]

$$\mathbf{H} = \sqrt{\frac{k}{k+1}}\bar{\mathbf{H}} + \sqrt{\frac{1}{k+1}}\tilde{\mathbf{H}} \tag{12}$$

where $k$ is the Ricean factor, and it is defined as the power ratio of the LoS component to the NLoS component. $\bar{\mathbf{H}}$ is a $K \times M$ deterministic matrix representing the LoS part while $\tilde{\mathbf{H}}$ is a $K \times M$ random matrix representing the NLoS part.

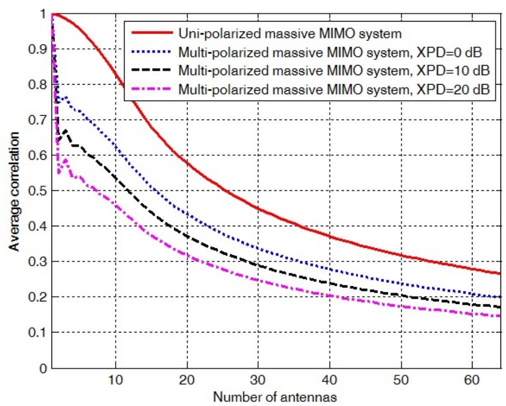



Fig. 3. Average correlation versus number of antennas with different XPD.

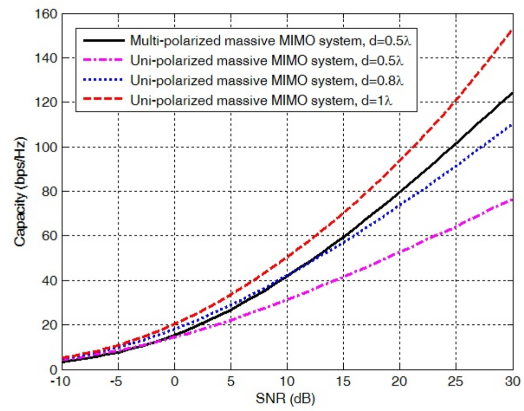



Fig. 4. Channel capacity versus SNR with different antenna spacing.

For scattering part, the correlated channel model can be modeled as Kronecker model

$$\mathbf{H}_c = \mathbf{R}_r^{1/2}\mathbf{H}_{\mathbf{i.i.d.}}\mathbf{R}_t^{T/2} \tag{13}$$

where $\mathbf{H}_{\mathbf{i.i.d.}}$ is a $K \times M$ matrix of independent and identically distributed (i.i.d.) zero mean complex-valued Gaussian random variables to describe the uncorrelated scattering channel with power balance. $\mathbf{R}_r$ is $K \times K$ receive correlation matrix and $\mathbf{R}_t$ is $M \times M$ transmit correlation matrix. $(\cdot)^T$ denotes the transpose operation. In addition, the power imbalance has to be considered in the multi-polarized massive MIMO systems. The power imbalance is caused by the polarization mismatch (i.e., the vertically polarized antennas have zero gain to the horizontally polarized direction signal and vice versa), and it can be described by a $K \times M$ polarization matrix

$$\mathbf{H}_p = \begin{bmatrix} \sqrt{1-\alpha} & \sqrt{\alpha} & \dots \\ \sqrt{\alpha} & \sqrt{1-\alpha} & \dots \\ \dots & \dots & \dots \end{bmatrix}. \tag{14}$$

Hence, the scattering matrix (or channel) including correlation and power imbalance is

$$\tilde{\mathbf{H}} = \mathbf{H}_p \odot \mathbf{H}_c \tag{15}$$

where $\odot$ is the corresponding elements multiplication. As for a fixed matrix (or channel), it is assumed that $\bar{h}_{XX}$ is 1 and $\bar{h}_{XY}$ is 0 in $\bar{\mathbf{H}}$ [4] because there are no polarization changes and rotations in LoS part. The generalized channel capacity can be computed as

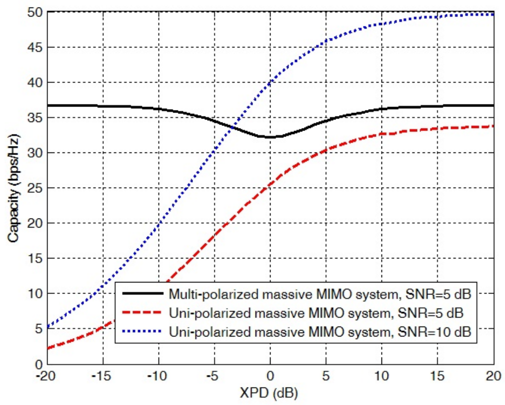
$$\mathbf{C} = \log_2[\det(\mathbf{I} + \frac{\gamma}{\mathrm{M}}\mathbf{H}\mathbf{H}^{\mathrm{H}})] \tag{16}$$

where $\mathbf{I}$ is the $K \times K$ identity matrix, $\mathbf{H}$ denotes the $K \times M$ channel matrix, and $\mathbf{H}^H$ is its conjugate transpose. $\gamma$ is the average signal-to-noise ratio (SNR) at each receiver branch.

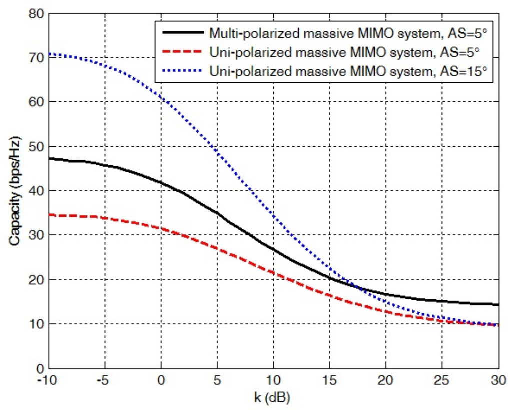
## III. Simulation Results and Analysis

Fig. 4 is the comparison of capacity between multi-polarized massive MIMO systems and uni-polarized massive MIMO systems with different antenna spacing. AS is set to be $5°$, XPD is set to be a mean value $10$ dB and Ricean k-factor can be equaled to $0$ dB to avoid any part becomes dominated. Both the number of transmitter antennas and receiver antennas are $64$. The larger antenna spacing makes the lower average correlation between antennas, hence resulting in a higher capacity. The multi-polarized massive MIMO systems have higher capacity compared to the uni-polarized massive MIMO systems even with smaller antenna spacing because of the reduction in the correlation between antennas. Therefore the multi-polarized antennas can be used in massive MIMO systems to enhance the system performance and realize the space efficiency. Also we can see that the multi-polarized massive MIMO systems do not always outperform uni-polarized massive MIMO systems. When SNR is very low, the multi-polarized massive MIMO systems have lower capacity because of the power loss resulting from the polarization mismatch and the reduction in the correlation is not enough to compensate for the power loss in the co-polarized component [18]. As for AS, it has the same effect on the systems performance as antenna spacing (i.e., larger AS results in higher capacity), which is not shown again.

Fig. 5 (a) shows the capacity changes as the XPD varies. From which we can see the capacity of uni-polarized massive MIMO systems decreases drastically as the XPD decreases. This is because the uni-polarized massive MIMO systems will lose most of the power due to the polarization mismatch at low XPD. While the multi-polarized massive MIMO systems always have cross-polarized antennas to receive the cross-polarized signals, therefore, the XPD has a slight effect on the multi-polarized massive MIMO systems. Fig. 5 (b) compares the channel capacity of multi-polarized massive MIMO systems with uni-polarized massive MIMO systems as $k$ changes. We can see that the channel capacity of both multi-polarized massive MIMO systems and uni-polarized massive MIMO systems decreases as $k$ increases. This is because the LoS part becomes dominant and the scattering part becomes weak as $k$ increases, which results in high correlation between antennas. While the multi-polarized massive MIMO systems




(a)




(b)

Fig. 5. (a) Channel capacity versus XPD (b) Channel capacity versus k-factor.

outperform the uni-polarized massive MIMO systems as $k$ increases because the multi-polarized massive MIMO systems have low correlation.

## IV. Conclusion

In this paper, a 3-D geometrical channel model is established for proposed point-to-point multi-polarized massive MIMO systems. The channel is modeled as a Ricean fading channel and the average correlation for the whole systems is defined to indicate the correlation degree of the systems. Simulation results show that using multi-polarized antennas in massive MIMO systems can help to reduce the correlation between antennas and can reduce the demand for large antenna spacing to realize the space efficiency. And the multi-polarized massive MIMO systems have better performance compared to the uni-polarized massive MIMO systems in many situations.

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